# Strings in $AdS_4 \times CP^3$, and three paths to $h(\lambda)$

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Work (and work in progress) with
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Puri, 7 January 2011

$$\Delta - \frac{J}{2} = \sqrt{\frac{Q^2}{4} + 4\, h(\lambda)^2 \sin^2 \frac{p}{2}}$$

Exact dispersion relation $E(p)$ for:

- Magnons of the spin chain for ABJM,

$$h(\lambda) = \lambda + b\lambda^3 + \ldots \qquad (\lambda \ll 1)$$

- String excitations in $AdS_4 \times CP^3$, $\qquad (\lambda \gg 1)$

$$h(\lambda) = \sqrt{\frac{\lambda}{2}} + c + \frac{d}{\sqrt{\lambda}} + \ldots \qquad c = -\frac{\log 2}{2\pi}$$

or possibly $c = 0$?

Will discuss three AdS/CFT tests which tell us about $c$.

*— Programme —*

1. Integrability and the AdS/CFT spectral problem
2. The new example of ABJM
3. One-loop energy corrections for spinning strings
4. ... and for giant magnons, using algebraic curves [June 2010]
5. Extension to the case $J < \infty$ [MA/IA/DB, i.p.]
6. The near-flat-space limit and its uses [MA/PS, i.p.]
7. And two loops?

# 1 The Spectral Problem for $\mathcal{N} = 4$ SYM

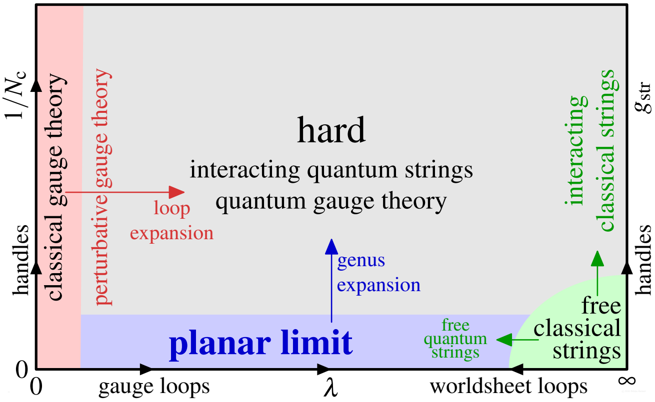
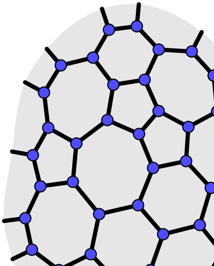



Figure from review [Beisert et. al. 2010]

In the strictly planar limit of AdS/CFT, we now know the spectrum of $\Delta$ for all $\lambda$.



Easiest case is $J = \infty$, where we have:

- Very long operators $O = \mathrm{Tr}(ZZZZZZZZ\ldots^{J})$ + impurities with $\Delta - J$ a spin-chain Hamiltonian:

$$\Delta - J = \sum_i \lambda \mathcal{H}_{i,i+1} + \lambda^2 \mathcal{H}_{i,i+1,i+2} + \lambda^3 \mathcal{H}_{i,i+1,i+2,i+3} + \ldots$$

- Strings with infinite $SO(6)$ angular momentum, thus decompactified worldsheet $X^\mu(\sigma \in \mathbb{R}, \tau = t)$ and semiclassical corrections, $\mathcal{O}(1/\sqrt{\lambda})$

- Asymptotic Bethe equations give the spectrum as solution of

$$\text{“}B(\Delta, \lambda) = 0\text{”}$$

connecting large and small $\lambda$.

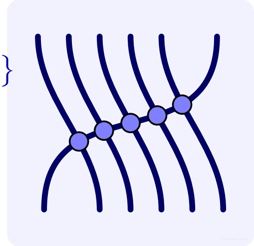
$J = \infty$ is easy because excitations can be widely separated:

- Dispersion relation $\quad E(p) = \sqrt{1 + \frac{\pi}{\lambda} \sin^2 \frac{p}{2}}$ for isolated particle,

  Energies are additive: $\quad E_{\{i\}} = \sum_i E(p_i)$

- Two-particle S-matrix $\quad S(p_i, p_i)$

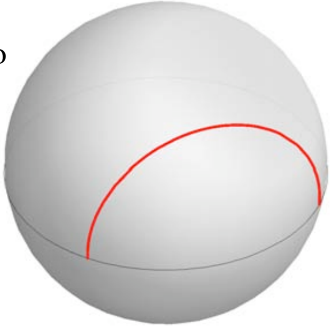
  Factorised scattering: $\quad S_{\{i\}\{j\}} = \prod_{ij} S(p_i, p_j)$



Bethe's Ansatz for N-particle state is a superposition $\{p_i\}$ constrained by $\psi(0, x_2, x_3 \ldots x_N) = \psi(J, x_2, x_3 \ldots x_N)$ on a circle of $J \approx \infty$ size.

Similar equations for $J < \infty$: "Thermodynamic Bethe Ansatz" / "Y-system"
[Gromov, Kazakov, Vieira] [Arutyunov, Frolov, Suzuki] 2009

Giant magnons are classical string solutions dual to spin chain magnons: [Hofman & Maldacena, 2006]



$$X^1 + iX^2 = e^{it}\left[\cos\tfrac{p}{2} + i\sin\tfrac{p}{2}\tanh(u)\right]$$
$$X^3 = \sin\tfrac{p}{2}\,\mathrm{sech}(u)$$

where $u = (x - t\cos\frac{p}{2})/\sin\frac{p}{2}$. Charges

$$E(p) = \Delta - J = \frac{\sqrt{\lambda}}{\pi}\sin\frac{p}{2}$$

Turning on 2nd charge $Q \sim \sqrt{\lambda}$ in the $X^3$-$X^4$ plane gives:

$$E(p,Q) = \Delta - J = \sqrt{Q^2 + \frac{\lambda}{\pi^2}\sin^2\frac{p}{2}}$$

"Dyonic giant magnon" in $\mathbb{R}\times S^3$ [Dorey] [Chen, Dorey, Okamura] 2006
dual to a bound state of $Q$ spin chain magnons.

# 2 ABJM and $AdS_4 \times CP^3$

ABJM [2008] is 3+1-dim. $\mathcal{N} = 6$ superconformal Chern-Simons theory.

Dual to M2-branes in $AdS_4 \times S^7/\mathbb{Z}_k$ , (also [BL & G, 2007-8-9] etc.)

KK reduction as $k \to \infty$ leads to IIA strings on $CP^3$.

Planar limit has 't Hooft coupling

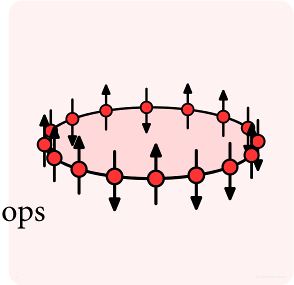
$$\frac{N}{k} = \lambda = \frac{R^4}{32\pi^2\alpha'^2}$$

New example of integrable AdS/CFT.

Almost everything can be copied across, with slight modifications.

Scalar fields are in $(N, \bar{N})$ of $U(N)$ (rather than adjoint), and the spin chain vacuum is

$$O = \mathrm{Tr}\left(Y_1 Y_4^\dagger\right)^J$$



Can excite even or odd chain, decoupled at leading order:

$$\Delta - \frac{J}{2} = \sum_i \left(\mathcal{H}_{i,i+2} + \mathcal{H}_{i+1,i+3}\right) + \text{ four loops}$$

Symmetries fix the exact dispersion relation:

$$\Delta - \frac{J}{2} = \sqrt{\frac{Q^2}{4} + 4\, h(\lambda)^2 \sin^2 \frac{p}{2}}$$

but leave the function $h(\lambda)$ unknown.

(True in $AdS_5 \times S^5$ too, but there $h(\lambda) = \lambda$ thanks to experiments and an argument from S-duality [Berenstein & Trancanelli, 2009])

At $\lambda \ll 1$, leading term is 2 loops. [Minahan & Zarembo, 2008]

Next term comes from 4 loops:

$$h(\lambda)^2 = \lambda^2 - 4\,\zeta(2)\,\lambda^4 + \dots$$

[Leoni, Mauri, **Minahan, Ohlsson Sax,** Santambrogio, **Sieg,** Tartaglino-Mazzucchelli 2010] (and earlier papers by bold names)

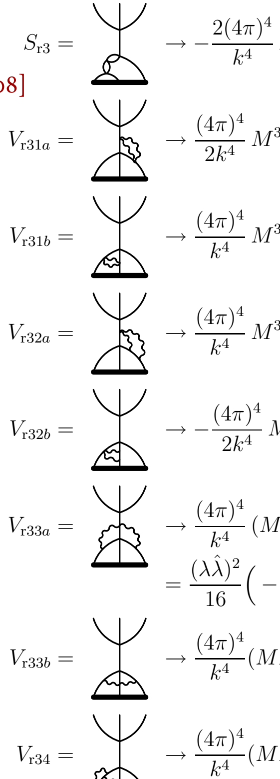
Their all-$\lambda$ guess is:

$$h(\lambda)^2 = \frac{1}{2\pi^2}\sum_{\pm} \pm(1 \pm 2\pi i\lambda)\log(1 \pm 2\pi i\lambda)$$

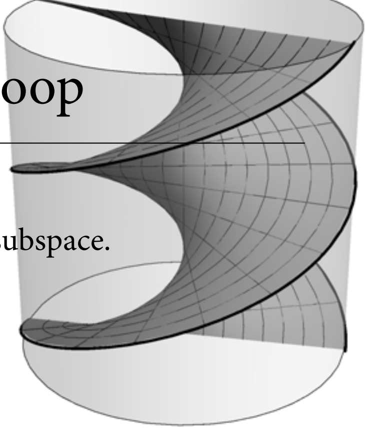
consistent with $c = 0$ when $\lambda \gg 1$:

$$h(\lambda) = \sqrt{\frac{\lambda}{2}} + c + \dots$$

(leading term from PP-wave ).



# 3 String Solitons at One Loop



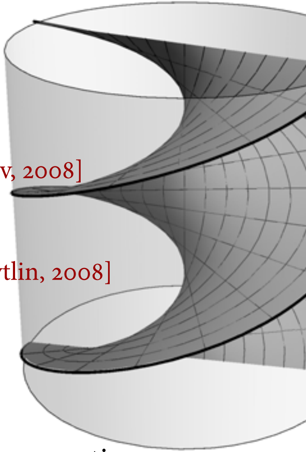
First example: folded spinning strings in $AdS_3$ subspace.

Classical solution $\phi = \tau,\ \rho = \sigma$,
with charges $\Delta - S = \sqrt{2\lambda}\log S$ when $S \to \infty$.

One-loop "disagreement":

$$\begin{aligned} \delta\Delta &= -3\frac{\log 2}{2\pi}\log S && \text{from } sl(2) \text{ Bethe equations} \\ &= -5\frac{\log 2}{2\pi}\log S && \text{explicit string calculation} \end{aligned}$$

[Gromov & Vieira] vs. [McLoughlin & Roiban] +
[Alday, Arutyunov, Bykov] + [Krishnan], 2008



Two resolutions:

- Modify the summation prescription, keeping $c = 0$ like $S^5$ case. [Gromov & Mikhaylov, 2008]
- Turn on $c = -\frac{\log 2}{2\pi}$, and keep naïve mode sum. [McLoughlin, Roiban, Tseytlin, 2008]

Summary:

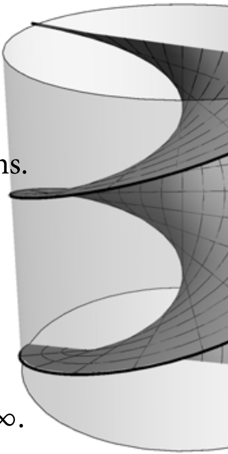
$$\frac{\Delta - S}{\log S} = 2h(\lambda) - 3\frac{\log 2}{2\pi} + o\left(\frac{1}{h}\right) \qquad \text{from } sl(2) \text{ Bethe equations}$$

$$= \sqrt{2\lambda} + \begin{cases} -5\dfrac{\log 2}{2\pi} & \\ & \text{using the} \\ -3\dfrac{\log 2}{2\pi} & \end{cases} \begin{matrix} \text{old sum} \\ \\ \text{new sum} \end{matrix} \text{ with } \begin{matrix} c_{\text{old}} = -\dfrac{\log 2}{2\pi} \\ \\ c_{\text{new}} = 0 \end{matrix}$$

String calculations are

$$\delta E = \sum_n \frac{\hbar}{2}\omega_n$$

Prototype is sine-gordon: compare one-soliton to no-solitons.
This tends to be very infinite ... but $(-1)^F$ will save us.



Modes are of course perturbations like this

$$X^\mu_{\text{classical}} + e^{-i\omega_n t}\delta X^\mu_n$$

becoming plane waves $e^{ikx-i\omega t\pm i\delta/2}$ as $x \to \pm\infty$.

In $AdS_5 \times S^5$, all of these modes have the same mass: $\omega^2 = k^2 + 1$.

But in $AdS_4 \times CP^3$, instead $\begin{cases} \omega^2 = k^2 + 1 & \text{heavy} \\ \omega^2 = k^2 + 1/2 & \text{light} \end{cases}$

($\exists$ subspaces radius $R$ and $R/2$)

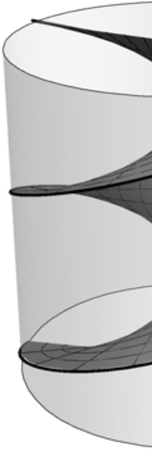
The two choices of cutoff are:

$$\delta E_{\text{old}} = \lim_{N\to\infty} \sum_{n=-N}^{N} \left( \omega_n^{\text{light}} + \omega_n^{\text{heavy}} \right)$$

$$\Rightarrow \; c = \frac{-\log 2}{2\pi}$$

$$\delta E_{\text{new}} = \lim_{N\to\infty} \left( \sum_{n=-N}^{N} \omega_n^{\text{light}} + \sum_{n=-2N}^{2N} \omega_n^{\text{heavy}} \right)$$

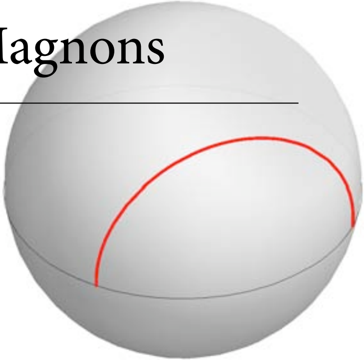
$$\Rightarrow \; c = 0$$



Heavy modes...

- are simply 4 of the 8 $\perp$ directions in space! (and fermions)
- do not appear in the Bethe ansatz
  (they are superpositions, or "stacks")
- are perhaps composite at one loop? [Zarembo, 2009]

# 4 Corrections for Giant Magnons



Compare one-loop corrections
to exact dispersion relation:

$$\Delta - \frac{J}{2} = \sqrt{\frac{Q^2}{4} + 4\, h(\lambda)^2 \sin^2 \frac{p}{2}}$$

$$= \sqrt{\frac{Q^2}{4} + 2\lambda\, \sin^2 \frac{p}{2}} + \frac{c\,\sqrt{8\lambda}\sin^2 \frac{p}{2}}{\sqrt{\frac{Q^2}{4} + 2\lambda\, \sin^2 \frac{p}{2}}} + \mathcal{O}\left(\frac{1}{\sqrt{\lambda}}\right)$$

An early result [Shenderovich, 2008] gave $c = 0$,
confusingly before [G&V]'s new sum.

Giant magnon in $CP^1$ is identical to $S^2$ case. [Gaiotto, Giombi, Yin, 2008]

But the dyonic version is new, [MCA, Aniceto, Ohlsson Sax, 2009]
explores $CP^2$ by turning on $\xi \neq \pi/2$, $\varphi_1 = \omega t + \ldots$:

$$\mathbf{z} = \begin{pmatrix} \sin\xi \, \cos(\vartheta_2/2) \, e^{i\varphi_2/2} \\ \cos\xi \, e^{i\varphi_1/2} \\ 0 \\ \sin\xi \, \sin(\vartheta_2/2) \, e^{-i\varphi_2/2} \end{pmatrix}$$

Other giant magnons in $RP^2$ (and dyonic $RP^3$) are superpositions of two elementary magnons. [Hollowod & Miramontes, 2009]

In principle we could compute $\delta X^\mu(x,t)$ from worldsheet solutions by hand (like $S^5$ case [Papathanasiou & Spradlin, 2007])
but it is less work to use power tools...

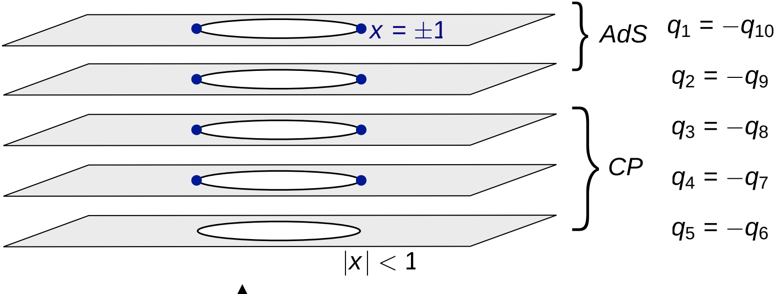
Use some integrable systems technology called the algebraic curve:

$$\text{Classical string solutions} \underset{\text{one-to-one}}{\longleftrightarrow} \text{Riemann surfaces}$$

Construction from Lax connection is like this:

$$M(x) = P \exp \oint d\sigma \, J_\sigma(x)$$

$$\mathrm{eig} M = \left\{ e^{ip_1(x)}, e^{ip_2(x)}, e^{ip_3(x)}, \ldots \right\}$$



Well-developed scheme for semiclassical perturbations:

- Add $\sqrt{\phantom{xx}}$ cut connecting sheets $(i, j)$
- at point $y$ solving $q_i(y) - q_j(y) = 2\pi n$
- with filling fraction $S_{ij} = \frac{g}{i\pi} \oint_{C_{ij}} dx \left(1 - \frac{1}{x^2}\right) q_i(x) = 1$

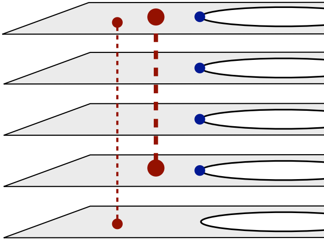
[Beisert, Kazakov, Sakai, Zarembo, 2005]
[Gromov, Vieira, 2007]

After constructing mode $\delta q_i(x)$,
you can read off its perturbation of the energy:

$$\delta\Delta = \Omega(y) = \omega$$

The you add all of these up...

Light polarisations $(i, j)$ connect to sheet 5(=6)
Heavy ones do not.



For giant magnons, this gives simple "off-shell" frequencies:

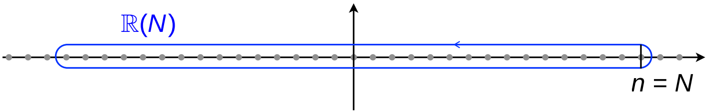
$$\Omega(y) = \frac{1}{y^2 - 1}\left(1 - y\frac{X^+ + X^-}{1 + X^+X^-}\right) \times \begin{cases} 1 & (i,j) \text{ light} \\ 2 & \text{heavy} \end{cases}$$

Not easy to find positions $x_n^{ij}$, hence "on-shell" frequencies $\omega_n^{ij} = \Omega(x_n^{ij})$.

Can still add them up, with some complex analysis: [Schäfer-Nameki 2006]

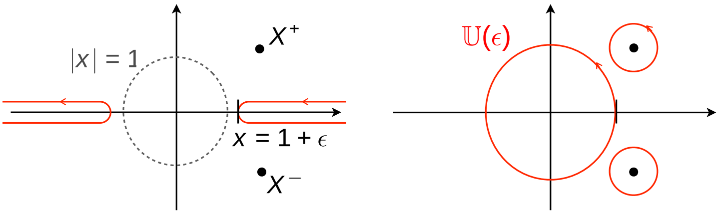
$$\begin{aligned} \delta E &= \frac{1}{2}\sum_n \Omega_{ij}(x_n^{ij}) \\ &= \frac{1}{4i}\oint_{\mathbb{R}} dn \sum_{ij}(-1)^{F_{ij}} \cot(\pi n)\Omega_{ij}(x_n^{ij}) \end{aligned}$$

*n* plane:




*x* plane:

*x* plane:




Using $q_i(x_n^{ij}) - q_j(x_n^{ij}) = 2\pi n$, write in $x$:

$$\delta E = \frac{1}{4i} \oint_{-\mathbb{U}} dx \sum_{ij} (-1)^{F_{ij}} \frac{q_i'(x) - q_j'(x)}{2\pi} \cot\left( \frac{q_i(x) - q_j(x)}{2} \right) \Omega_{ij}(x)$$

For now ($J = \infty$) can ignore other contour components.

But we can't ignore details of the cutoff
$|n| < N$, which is $|x| > 1 + \epsilon$ ...

New sum is simplest: $x_{2N}^{\text{heavy}} \approx x_N^{\text{light}}$, thus cut off at same $x = 1+\epsilon$ for both:

$$\delta E_{\text{new}} = \lim_{\epsilon\to 0} \sum_{ij} \oint_{\mathbb{U}(\epsilon)} dx \, \frac{(-1)^{F_{ij}}}{-4i} \frac{q_i'-q_j'}{2\pi} \cot\left(\frac{q_i-q_j}{2}\right) \Omega_{ij}(x)$$

$$= 0$$

Old sum is more work, $x_N^{\text{heavy}} \approx 2x_N^{\text{light}}$ so

$$\delta E_{\text{old}} = \lim_{\epsilon\to 0} \left\{ \sum_{ij\,\text{light}} \oint_{\mathbb{U}(\epsilon)} dx + \sum_{ij\,\text{heavy}} \oint_{\mathbb{U}(2\epsilon)} dx \right\} \frac{(-1)^{F_{ij}}}{-4i} \frac{q_i'-q_j'}{2\pi} \cot\left(\frac{q_i-q_j}{2}\right) \Omega_{ij}(x$$

$$= \frac{-\log 2}{2\pi} 2 \sin \frac{p}{2}$$

Dyonic case: $\delta E_{\text{old}} = \frac{-\log 2}{2\pi} \frac{\sqrt{8\lambda} \sin^2 \frac{p}{2}}{\sqrt{\frac{Q^2}{4} + 2\lambda \sin^2 \frac{p}{2}}}$. [MCA, Aniceto, Bombardelli, 2010]

All consistent with previous AdS results...

(Review [Klose, 2010] latest [APGHO,2011] )

Arguments about cutoff prescriptions:

**Old**:

$$\sum^{N}(\text{light} + \text{heavy})$$

$$c = \frac{-\log 2}{2\pi}$$

Easiest in worldsheet calculations.

Equivalent to hard energy cutoff: $\omega_N \propto N \propto \Lambda$ same for both types. (Freq. w.r.t. AdS time.)

In the spectral plane,

$$\int_{2\epsilon} \text{heavy} + \int_{\epsilon} \text{light}$$

**New:**

$$\sum^{2N} \text{heavy} + \sum^{N} \text{light}$$

$$c = 0$$

Because heavy mode is composite? $\omega_{2N}^{\text{heavy}} \approx \omega_N + \omega_N$.

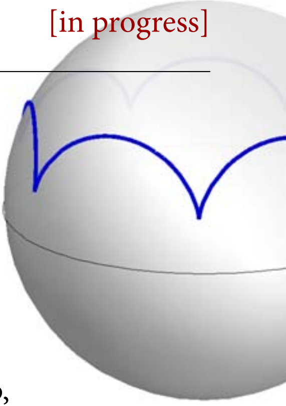
Easier to match all=$\lambda$ guess?

$$\int_{\epsilon}(\text{light} + \text{heavy})$$

... hence easiest in algebraic curve calculations.

# 5 Giant magnons at $J < \infty$

[in progress]



Corrections are organised like this:

$$E = \sum_{m,n=0,1,2\ldots} a_{m,n} \left(e^{-\Delta/\sqrt{2\lambda}}\right)^m \left(e^{-2\Delta/E}\right)^n$$

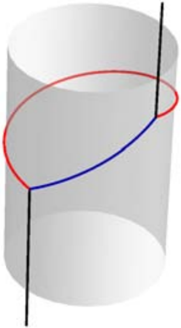
- $a_{0,0} = E_{\text{class.}} + \delta E$ is the case $J = \infty$ from before.
- Corrections $a_{0,1}$ are $F$-terms, zero classically.
- Corrections $a_{1,0}$ are $\mu$-terms, classical + one-loop, so we can make a comparison:

$$\begin{aligned} a_{0,1} e^{-2\Delta/E} &= h(\lambda)\, a_{\text{class.}}(p,Q)\, e^{-2\Delta/E_0(h,p,Q)} + a_{\text{subl.}} e^{-2\Delta/E_0} + \mathcal{O}\left(\frac{1}{h}\right) \\ &= \sqrt{\frac{\lambda}{2}}\, a_{\text{class.}}(p,Q)\, e^{-2\Delta/E_0\left(\sqrt{\lambda/2},p,Q\right)} + \delta E^\mu + \mathcal{O}\left(\frac{1}{\sqrt{\lambda}}\right) \end{aligned}$$

Gauge side: Lüsher terms, which are wrapped Feynman diagrams:

F-term:

$\mu$-term:



F-terms: computed by [Bombardelli & Fioravanti, 2008], classically zero.

$\mu$-terms: Unsolved (order-of-limits?) issues for single elementary magnon [Lukowski & Ohlsson Sax, 2008] [Bombardelli & Fioravanti, 2008]

For a bound state (dyonic magnon):

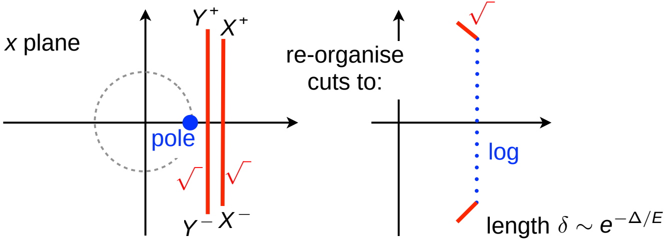
- classical $\mu$-term OK, ($S^5$ case: [Hatsuda & Suzuki, 2008])
- one-loop term not certain...

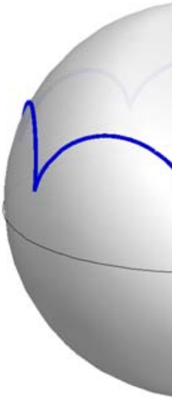
Classical string solutions:

- Map to kink train in sine-gordon [Okamura & Suzuki, 2006] or construct $X^\mu(\sigma,\tau)$ directly.

- Algebraic curve: this is the more natural case!

  Giant magnon is a two-cut solution:



Semiclassical calculation is an expansion in $\delta^2$ of previous integral $\int_{\mathbb{U}(\epsilon)} dx \ldots$, plus some discrete terms. ($S^5$ case: [Gromov, Schäfer-Nameki, Vieira, 2008])

Old vs. New:

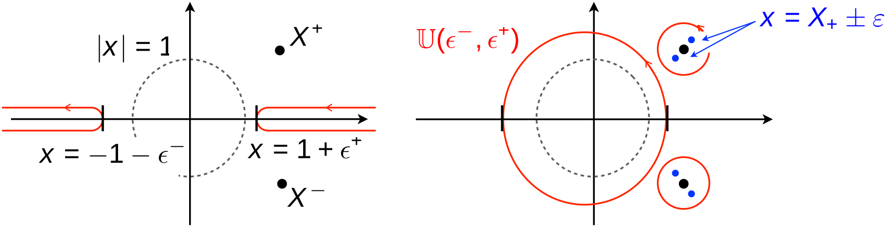
- Earlier, I claimed "old" ≡ "physical": all modes to same energy.

  Here, "old" leads to divergences, but "physical" like this works:

$$\delta E_{\text{phys}} = \lim_{\Lambda\to\infty} \sum_{ij} (-1)^{F_{ij}} \sum_{\left|\omega_n^{ij}\right|<\Lambda} \frac{1}{2}\omega_n^{ij} \qquad = \lim_{\Lambda\to\infty} -\frac{1}{4i}\sum_{ij}(-1)^{F_{ij}} \oint_{\mathbb{U}(\epsilon^-_{ij},\epsilon^+_{ij})} dx$$

  Needs cutoffs for every polarisation: $\Omega_{ij}(-1-\epsilon^-_{ij}) = \Omega_{ij}(1+\epsilon^+_{ij}) = \Lambda$

$x$ plane:



Cut structure is different:

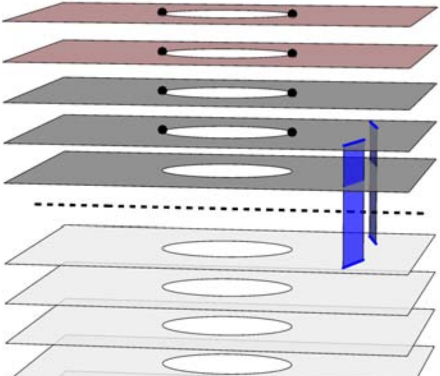
- In $S^5$ case, cuts always connect sheets $p_{\tilde{2}}(x)$ and $p_{\tilde{3}}(x) = -p_{\tilde{2}}(x)$, allowing ansatzae of the form

$$p'(x) = \frac{1}{\sqrt{(x-X^+)(x-Y^+)(x-X^-)(x-Y^-)}}\left(K + \frac{\alpha f(1)}{x-1} +\right.$$

- But for $CP^3$ they connect $q_4(x)$ and $q_6(x)$...

The $RP^3$ magnon is like $S^5$ in this regard, and for this we can compute both "new" $c = 0$ and "physical" $c = -\frac{\log 2}{2\pi}$.

Both match match Lüscher corrections from [Bombardelli & Fioravanti, 2008].



# 6 The Near-Flat-Space Limit [in progress]

Intermediate limit: [Maldacena & Swanson, 2006]

- BMN: $p_{ws} \sim 1/\sqrt{\lambda}$
- Near-flat-space: $p_{ws} \sim 1/\lambda^{1/4}$
- Magnons: $p_{ws} \sim 1$

Write the dispersion relation as follows:

$$
\begin{aligned}
E^2 &= \frac{1}{4} + 4\,h(\lambda)^2 \sin^2 \frac{p_{ws}}{2} \\
&= \frac{1}{4} + p_1^2 + \left[ \frac{c\,p_-^2}{\sqrt{2\lambda}} - \frac{p_-^4}{96\lambda} \right] + \mathcal{O}\left( \frac{1}{\sqrt{\lambda}} \right)
\end{aligned}
$$

$$p^2 = E^2 - p_1^2 \quad = \frac{1}{4} + \quad \left[\delta m^2\right]$$

Thus mass corrections to the propagator will teach us about $c$:

$$G_2(p) = \frac{i}{p^2 - \frac{1}{4}} + \frac{i}{p^2 - \frac{1}{4}} \mathcal{A} \frac{i}{p^2 - \frac{1}{4}} + \ldots = \frac{i}{p^2 - \frac{1}{4} - \delta m^2}$$

Near-BMN Lagrangian computed by [Sundin, 2009], from coset model.

Taking a large boost of this $p_- \sim \lambda^{1/4} \to \infty$, $p_+ \sim \lambda^{-1/4} \to 0$
and shifting fields to get canonical $\mathcal{L}_2$
leads us to the following theory:

Quadratic: $$\frac{1}{2}\partial_+ y\partial_- y + \frac{1}{2}\partial_+ z_i\partial_- z_i + \frac{1}{4}\partial_+\omega_\alpha\partial_-\bar{\omega}^\alpha + \frac{1}{4}\partial_-\omega_\alpha\partial_+\bar{\omega}^\alpha - \frac{1}{2}\big(y^2 + z_i^2\big) - \frac{1}{8}\omega_\alpha\bar{\omega}^\alpha$$
$$+\frac{i}{2}\big(\bar{\psi}_{+a}\overleftrightarrow{\partial}_-\psi^{+a} + \bar{\psi}_{-a}\overleftrightarrow{\partial}_+\psi^{-a}\big) - \frac{i}{2}\big((s_-)^a_\alpha\partial_+(s_-)^\alpha_a + (s_+)^a_\alpha\partial_-(s_+)^\alpha_a\big)$$
$$+\frac{1}{2}\big(\bar{\psi}_{-a}\psi^a_+ + \bar{\psi}_{+a}\psi^a_-\big) + i(s_+)^a_\alpha(s_-)^\alpha_a.$$

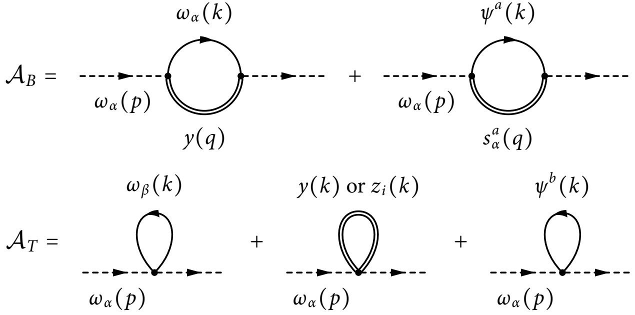
Cubic: $$-\mathscr{L}_3 = \frac{i}{4}(s_-)_{a\alpha}\big(\partial_+\bar{\psi}^a_-\partial_-\omega^\alpha - \partial_+\psi^a_-\partial_-\bar{\omega}^\alpha\big) + \frac{3i}{16}(s_-)_{a\alpha}\big(\bar{\psi}^a_-\omega^\alpha - \psi^a_-\bar{\omega}^\alpha\big) \quad (2$$
$$-\frac{i}{4}\partial_+(s_-)_{a\alpha}\big(\bar{\psi}^a_-\partial_-\omega^\alpha - \psi^a_-\partial_-\bar{\omega}^\alpha\big) + \frac{i}{2}(s_+)_{a\alpha}\big(\partial_-\bar{\psi}^a_-\omega^\alpha - \partial_-\psi^a_-\bar{\omega}^\alpha\big)$$
$$-\frac{1}{4}\partial_-(s_-)_{a\alpha}\big(\bar{\psi}^a_+\omega^\alpha + \psi^a_+\bar{\omega}^\alpha\big) + \frac{1}{2}\big(\partial_-\bar{\psi}_{-a}\psi^b_+ + \bar{\psi}_{+a}\partial_-\psi^b_-\big)Z^a_b - \frac{i}{8}y\omega_\alpha\overleftrightarrow{\partial}_-\bar{\omega}^\alpha$$
$$+\frac{i}{2}\big(\bar{\psi}_{-a}\partial_+\psi^b_- - \partial_+\bar{\psi}_{-a}\psi^b_-\big)\partial_- Z^a_b$$

Quartic: $$\mathscr{L}_{BB} = -\frac{1}{8}\big(z_i^2 - y^2 - \frac{1}{4}\omega_\alpha\bar{\omega}^\alpha\big)\big((\partial_- y)^2 + \partial_-\omega_\alpha\partial_-\bar{\omega}^\alpha + (\partial_- z_i)^2\big)$$

$$-\mathscr{L}_{BF} = \qquad (2.16)$$
$$-\frac{i}{8}y^2(s_-)_{a\alpha}\partial_-(s_-)^{a\alpha} - \frac{i}{8}(s_-)_{a\alpha}(s_-)^a_\gamma\Big(\partial_-\omega^\alpha\,\bar{\omega}^\gamma - \omega^\alpha\partial_-\bar{\omega}^\gamma\Big) - \frac{i}{32}\Big(\partial_-\bar{\psi}_-\,\psi_- - \bar{\psi}_-\,\partial_-\psi_-\Big)\omega\,\bar{\omega}$$
$$+\frac{i}{8}\Big(\bar{\psi}_-\,\psi_-\partial_-\omega\,\bar{\omega} + \bar{\psi}_-\,\partial_-\psi_-\,\omega\,\bar{\omega} - \bar{\psi}_-\,\psi\,\omega\,\partial_-\bar{\omega} - \partial_-\bar{\psi}_-\,\psi\,\omega\,\bar{\omega} - \frac{1}{2}(s_-)_{a\alpha}\partial_-(s_-)^{a\alpha}\,\omega\,\bar{\omega}\Big)$$
$$+\frac{3}{16}y(s_-)_{a\alpha}\big(\psi^a_-\partial_-\bar{\omega}^\alpha + \bar{\psi}^a_-\partial_-\omega^\alpha\big) - \frac{i}{8}\bar{\psi}_{-a}\psi^b_- Z^a_c\overleftrightarrow{\partial}_- Z^c_b + \frac{i}{8}(s_-)_{a\alpha}\partial_-(s_-)^{a\alpha}z_i^2$$
$$-\frac{1}{4}y(s_-)_{a\alpha}\partial_-(s_-)^\alpha_c Z^{ca} + \frac{i}{4}(s_-)_{a\alpha}\big(\partial_-\bar{\psi}_{-c}\omega^\alpha Z^{ca} - \partial_-\psi^b_-\bar{\omega}^\alpha Z^a_b\big) + \frac{i}{8}(s_-)_{a\alpha}(s_-)^\alpha_b Z^a_d\partial_- Z^{db}$$
$$+\frac{i}{8}\partial_-(s_-)_{a\alpha}\big(\bar{\psi}_{-c}\omega^\alpha Z^{ca} - \psi^b_-\bar{\omega}^\alpha Z^a_b\big) - \frac{i}{8}y^2\bar{\psi}_-\overleftrightarrow{\partial}_-\psi_-.$$

$$\mathscr{L}_{FF} = -\frac{1}{8}\big(\bar{\psi}_-\cdot\psi_-\big)^2 - \frac{i}{8}\partial_-\big(\bar{\psi}_-\cdot\psi_-\big)\big(\bar{\psi}_-\cdot\psi_+ - \bar{\psi}_+\cdot\psi_-\big)$$
$$-\frac{i}{4}\big(\partial_-\bar{\psi}_-\cdot\psi_-\bar{\psi}_-\cdot\psi_+ - \bar{\psi}_-\cdot\partial_-\psi_-\bar{\psi}_+\cdot\psi_-\big) - \frac{i}{2}\bar{\psi}_-\cdot\psi_-\big(\partial_-\bar{\psi}_-$$
$$-\frac{1}{8}\partial_+\big(\bar{\psi}_-\cdot\psi_-\big)\partial_-\big(\bar{\psi}_-\cdot\psi_-\big) - \frac{1}{4}\bar{\psi}_-\cdot\psi_-\big(\partial_+\bar{\psi}_-\cdot\partial_-\psi_- + \partial_-\bar{\psi}_-$$
$$+\frac{1}{2}\big(\partial_+\bar{\psi}_-\cdot\psi_-\partial_-\bar{\psi}_-\cdot\psi_- + \bar{\psi}_-\cdot\partial_+\psi_-\bar{\psi}_-\cdot\partial_-\psi_-\big) + \frac{1}{16}\Big\{(s_-)^\alpha_c$$
$$-(s_-)^\alpha_c\partial_+\bar{\psi}^a_-\psi^c_-\partial_-(s_-)_{a\alpha} - \partial_-(s_-)^\alpha_c\bar{\psi}^a_-\partial_+\psi^c_-(s_-)_{a\alpha} + \partial_+(s_-)^\alpha_c$$
$$+\partial_-(s_-)^\alpha_c\bar{\psi}^a_-\psi^c_-\partial_+(s_-)_{a\alpha} + (s_-)^\alpha_c\partial_-\bar{\psi}^a_-\partial_+\psi^c_-(s_-)_{a\alpha} + (s_-)^\alpha_c\partial_+$$
$$-2(s_-)^\alpha_c\bar{\psi}^a_-\partial_-\psi^c_-\partial_+(s_-)_{a\alpha} - 2\partial_+(s_-)^\alpha_c\partial_-\bar{\psi}^a_-\psi^c_-(s_-)_{a\alpha} + \partial_+(s_-$$
$$+\frac{i}{}\Big\{ -\partial_-(s_-)^\alpha_c\bar{\psi}^a_+\psi^c_+(\blacktriangle)_{a\alpha} + (s_-)^\alpha_c\bar{\psi}^a_+\psi^c\,\partial_-(s_-)_{a\alpha}\Big\} + \frac{1}{}(s_-$$

Diagrams for correction to light boson $\langle \bar{w}_\alpha w^\beta \rangle = \delta_\alpha^\beta \dfrac{2i}{p^2 - 1/4}$ are:

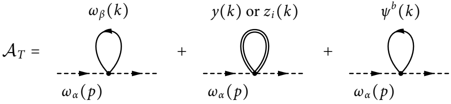



Bubble diagrams always contain both heavy and light,

so there is no way for the cutoff to discriminate?

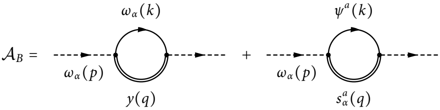
It is easiest to use dimensional regularisation...

For the tadpoles things are perfect:




$$\rightarrow \quad \delta m^2 = \frac{1}{\sqrt{2\lambda}} \frac{-\log 2}{2\pi}$$

But for the bubbles...




$$\rightarrow \quad \delta m^2 = \frac{1}{\sqrt{2\lambda}} \frac{1}{16\pi}$$

Also check other modes $\langle \bar{z}_i z_j \rangle$, $\langle \bar{\psi}_a \psi^b \rangle$, ...
and the S-matrix... [Klose & Zarembo, 2007]

$S^5$ case: [Klose, Minahan, Zarembo, McLoughlin, 2007]

# 7 Two-Loops?

We've discussed essentially two kinds of one-loop calculation.
Both kinds done in $AdS_5 \times S^5$ to two loop accuracy:

- Soliton energy corrections:
  Three papers and three years? [Roiban & Tseytlin, 2007]
  [Giombi, Ricci, Roiban, Tseytlin, Vergu, 2010]
- Near-flat-space:
  One sunset diagram, half a page!
  [Klose, Minahan, Zarembo, McLoughlin, 2007]

There is also an all-loop argument that $h(\lambda) = \lambda$, using S-duality, which fails for $AdS_4 \times CP^3$. [Berenstein & Trancanelli, 2009]

One further complication: relation $N/k = \lambda = R^4/32\pi^2\alpha'^2$ gets modified, starting at two loops $\lambda \gg 1$. [Bergman Hirano, 2009]

# The End.

With thanks to Olof, Nikolay, Shiraz, Valentina,

and collaborators Diego, Ines, Per.